Traditional teaching methods are inadequate to effectively educate a changing student population. The effective mathematics teacher of the 21st century must have a ready grasp of both basic and advanced mathematical concepts so as to be able to educate the advanced and the remedial student, a functional understanding of technology, be adept at instructional methods, have a ready knowledge of the applications of mathematics in numerous fields and disciplines, and be able to establish a relationship with the student, so as to create an atmosphere of trust and friendly study environment.

Issues

As technology changes rapidly during recent years, and especially currently, the whole world has begun focusing on wireless technology as the wave of the future — the on-ramp to the "information super highway." This all started, of course, with the invention of the computer. Clearly the computer has changed our society, just as the industrial revolution changed the world centuries ago. We can reasonably expect a similar revolution will occur in our society, especially in our education. The new education system for the 21st century will emphasize and require problem solving abilities and more math-oriented skills and, of course, computer literacy.

The great technology boom of the past ten years has dramatically changed the way we work, study, and play. What was originally thought of as little more than a technological wizard’s toy, the Internet, has expanded into a worldwide informational network which promises to revolutionize our lives in ways that we have not thought of yet. The Internet is changing the lives of our kids the way that television changed our lives decades ago, only the kids now are actually using their "brains" (Hardie, 1998).

Now we are able to shop online without leaving the comfort of our homes. We are able to write checks, pay bills, track the stocks, and manage finances. Students are able to sign up for courses
through the Internet, check their grades and do research papers through the abundance of reference
materials available on the Internet. We can even book airline tickets, can buy cars through 2700
accredited car dealers nationwide, we can shop at stores in other states, and even other countries. Smart
consumers can research their purchases and compare prices before buying (Hardie, 1998). Somewhat
more ominous, students are also able to buy term papers off the Internet for their classes.

Facing such incredible changes in technology, how do we, as educators in mathematics, adapt to the
changes as a whole in education?

Ironically, given that mathematics is the very foundation of technology, mathematics education has
been one of the most resistant to change (King, 1992; Malone, Rennie, Venville, & Wallace, (1998).
Perhaps this is because of the mathematician’s desire for preciseness, giving rise to a loathing for
change without certainty of success (Blume, 1991). Perhaps too many mathematics instructors believe
the first of the great lies of mathematics, "that there is only one way to get the answer," and, hence, any
change will produce wrong answers, and weaken the mathematics curriculum (Burrill, 1998; Amdahl,
& Loats, 1996).

At the Budapest meeting (ICME-6) in 1988, we already debated the problems of mathematics
education for the 21st century. Here was a portion of Chang’s address at one of the sessions of
Congress (Chang, 1988).

"...The retraining of re-educating of the current professionals in order to upgrade their
skills is one of the most important tasks facing us today. The new education system for
the next century will emphasize and require problem-solving abilities and more
mathematics-oriented skills. At the base of this new system will be the computer. The
personal computer will challenge designers of school programs to take advantage of the
simulation and graphics capacity, as well as the "word processing" abilities of modern
technology. The emerging system will be a cradle-to-grave system with a strong shift,
especially for younger children, from school-based formal learning to home-based
education."

"At present only primitive experiments in computer-assisted learning are available.
However, these will expand rapidly during this next decade to set the stage for
computer-based learning, which will then become central to our learning, just as books
and printed materials were central to our education systems a decade ago. Hence, the
education early in the next century will probably be based on the computer and other
electronic-information sources (ICIUT, 1983)."

This was the prediction of 12 years ago, during that time we were just adjusting to the computer
assisted learning programs and other electronic-information devices, such as graphing calculators. The
creation of the "information superhighway" and Internet was definitely not in our original thinking at
all. We had no way to predict the rapid development of information systems, and the creation of such
terms as e-mail, e-commerce and e-shoppers, e-privacy, on-line education and learning, and certainly
not the recent "digital graduates."

Disregard, for a moment, the rapid changes in technology. At the ICME-8 meeting in Seville, Spain in
1996, Chang suggested "three concerns" must be addressed in his "A Comparable Study of Teaching

"... How to teach mathematics effectively to students who are having difficulty in
learning to face this kind of massive technological revolution.
How to improve the teaching techniques of today’s teachers with this revolution, as well as their knowledge of subject matter.

How to train our future teachers to utilize the full range of scientific activities and materials to satisfy tomorrow’s new curriculum.

Effective Teachers

The solution to these concerns resides in the hands of our teachers. In the midst of the kind of radical change brought about by this sudden technological advancement it is always tempting to adopt extreme positions. Some instructors attempt to continue teaching their courses the way they have always taught, treating technological learning aids as useless toys, or, at best, irrelevant. Others embrace every teaching gimmick available, sacrificing understanding of the subject matter for an attractive, high-tech facade.

A moderate approach is needed. The instructor who neglects a technological understanding is unable to relate to his or her students for whom technology is a fundamental part of the way they understand. Too, regardless of technological advances, certain skills are still necessary to be an effective mathematics teacher. These five characteristics of an effective mathematics teacher for the twenty-first century were gleaned from a two-year mathematics in-service training program called the Mat-Su Math Co-op (1990-1992).

- A ready grasp of both basic and advanced mathematical concepts, so as to be able to educate the advanced pupil and remedy the slow learner’s grasp of mathematics,
- A functional understanding of technology, so as to be able to utilize computers as research tools, and install, operate, and train students in the use of mathematics tutorial programs while also recognizing the disadvantages of electronic learning aids,
- Adeptness at instructional methods so as to be able to enable and inspire students to learn,
- A ready knowledge of the applications of mathematics in numerous fields and disciplines,
- The ability to establish a relationship with the student so as to create an atmosphere of trust and understanding.

Revisiting the Mat-Su Math Co-op Project

As technology advances "rapid computation" and "find the quick answer" will not be sufficient to prepare our students to maintain a functional mathematical literacy for survival in the next century (Hyde, 1989). This view was also previously expressed by the National Council of Supervisors of Mathematics (NCSM) in 1977, and endorsed by the Mathematical Association of America (MAA) in 1979 and, in 1980, by the National Council of Teachers of Mathematics (NCTM).
NCSM position statements were revised again in 1989 to incorporate twelve components of essential mathematics with a reaffirmation of the importance of each in the next century. These include: problem solving, communicating, mathematical ideas, mathematical reasoning, applying mathematics to everyday situations, alertness to reasonableness of results, estimation, appropriate computational skills, algebraic thinking, measurement, geometry, statistics, probability, and climate for learning.

The Mat-Su Math Co-op Project, an Eisenhower Grant from 1990-1992, was an in-service education grant designed to train forty teachers from the Matanuska-Susitna School District in order to improve their mathematics knowledge and teaching techniques in the above areas with the intent of returning those teachers to their school to train and assist other teachers, and, of course, better educate their students.

The project consisted of seminars and workshops during regular semesters, plus a four day summer session in early June. The format and instructional mode used included lecture, demonstrations, group discussions, lab experimentation, and group project development activities. Topics were intended to implement the twelve components of mathematics suggested by the NCSM (1989).

It was found that teachers, particularly in secondary school, vitally needed a strong grasp of both basic and advanced mathematical concepts. In the past most pre-service teacher training in mathematics was specific to what they would be required to teach. Now teacher pre-service training institutions need to emphasize math maturity, and to develop a well-rounded knowledge of the subject. Without this background, teachers may be able to handle isolated mathematical knowledge and skill, but will not be able to develop an in-depth understanding of the subject that goes beyond what they will be expected to teach.

If a teacher cannot give students satisfactory answers, or avoids solving certain types of problems, it creates an attitude of confusion and hatred toward mathematics. As King (1992) states of the secondary school teacher who never developed math maturity:

". . . We saw signs of his ignorance even as he taught us. Mostly they showed through his fumbling and fearful responses to elementary questions that he could not answer: Why is it really that the product of two negative numbers is positive? How do you know those two bisectors intersect inside the triangle? What is the next number after 1/2?"

His reaction to each exposure of his ignorance was always the same. First came the fumbling stage where he tried to talk the question away by saying whatever popped into his head. . . . When that failed, as it invariably did, and he saw that it had failed, he became fearful. And when we saw the fright flash in his eyes we knew that this subject — mathematics — terrified our teacher deep down exactly as it terrified us."

King goes on to state that this situation leads to students’ perception of mathematics as something which can only be "endured," never understood or enjoyed. Even worse, students who struggle with mathematics are actually encouraged to drop out of the mathematics sequence.

K - 5 grade teachers

The K-5 teachers wanted to know more algebra. They wanted to have plenty of real life applications for their own grade level. They needed to know how to actively involve students in solving meaningful mathematics problems that build upon their experiences within branches of mathematics, and between
mathematics and other subjects. The goal of this being to enable their students to view mathematics as an instrument interwoven within every aspect of their lives.

**Problem-solving:** The geometry of various shaped polygons was a very popular lesson with the teachers we trained. Intuitively, they were able to realize the various shapes in the classroom, identified the similar triangle and popular square window, their beauty and their usefulness. We also emphasized geometric shapes in nature and the environment, and discussed the concepts of symmetry and balance.

They also needed examples of problem-solving situations for their grade level, especially that they would be able to use it, not only for the computational skills, but also for the cross-curricular activities. For example, using the study of the honeycomb and honeybee, the teacher can explain geometric shapes and explain why and how bees create honey, the life of the honeybee, that bees’ honeycombs are made of hexagonal shapes because the hexagon provides the largest volume for the least amount of material used while also providing a great deal of structural strength, the social nature of honey bees, how the bees transport honey, etc. The teacher can even go on to discuss honey in human society, including production and distribution. This gives the teacher the opportunity to involve every student in the class to study the whole unit of the honeybees, utilizing mathematics, social sciences, biological science, nutrition, and economics, involving pricing, shipping, packaging, and marketing, with each student working on a separate portion of the project. All this can be taught using one example, helping pupils to understand how mathematics is inherent in the various sciences. The honeybee example is often best used for the 5 - 8 grade levels, but simplified explanations may be used for K - 5 students.

**Math anxiety:** During the first meeting of the Math Co-op, several kindergarten and lower grade-level teachers asked why they need to study algebra if they already knew how to count, referring to their students’ limited needs. Numerous participants were very frustrated by simple factoring problems. They exhibited fear in solving story problems. Many participants received no advanced mathematical training beyond high school. The lower grade-level teachers needed tailor-made programs to teach simple addition and multiplication, plus models for real-life situations.

Math anxiety was a serious problem among the K-5 teachers. Most of the participants in this group lacked confidence, and hence interest, in learning more advanced topics involving mathematical induction, probability and statistics, logic, graph theory, as well as advanced algebraic factoring and problems.

**Technology use:** The early grade teachers would not introduce the use of calculators until third or fourth grade. The idea of using pencil and paper for computation for their earlier grade students was essential. This was agreed upon by all participants. In the fourth and fifth grades the teachers allowed students limited use of calculators. Of course, in a regular class, some used paper and pencil exclusively, while others used a combination of both. All agreed that the students must have mastered their computational skills with pencil and paper before going on to use calculators.

**6 - 12 grade teachers**

The 6-12 grade teachers wanted more algebraic applications and examples of real-life problem-solving situations. They enjoyed the set applications, mathematical induction, and various types of algebraic examples. They also liked to explore some topics in discrete mathematics, such as logic, set theory, graph theory, probability and statistics, the four-colored problem, and ring and group theory. A few participants wanted to explore topics in high-level mathematics, computer programming, and how the
use of computers affected classroom instruction. They were interested in using computers for drill and practice, tutorials, demonstrations, simulations, and learning how to use the computers to analyze data, develop generalizations, and even apply techniques in the program for problem solving (Fitting, 1983).

An example of problem-solving came in this problem (Figure 1):

![Figure 1.](image)

This problem-solving example created much debate during one of the problem-solving sessions. Participants utilized Polya’s four-step procedure (1971) for solving this problem. After discussion, the teachers came up with the following solution.

a. Understand the problem.

1. What is the meaning of the square?

2. How do we count them?

![Figure 2.](image)

b. Devise a plan.

1. Start from the beginning.

2. Look for a pattern.

c. Carry out the plan.

From the pattern established the teachers saw that $1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 1 + 4 + 9 + 16 + 25 = 55$, so that there are 55 different squares from the 5...
x 5 square. Therefore, the number of squares inside a N x N square is $1^2 + 2^2 + 3^2 \ldots + (N-1)^2 + N^2$.

d. Look back.

The participants were encouraged to look for other methods, and to check their solutions.

6 - 8 grade teachers’ specific needs: The needs of the middle school teachers were to increase their mathematical abilities. They wanted self-contained type models to aid them in teaching percentage and ratio problems.

9 - 12 grade teachers’ specific needs: The high school teachers preferred more advanced topics in modern algebra and discrete mathematics. They also wanted to teach in a manner in which they will be able to integrate classroom activities in a real-world application via problem solving, manipulatives, and electronic learning aids. Another concern was the need to prepare their students for college level mathematics.

Group sessions

Because of the difference in mathematical skills between teacher of different grade levels, we had both separate and group sessions. One of the goals of the group sessions was to increase communication between instructors of different grade levels. This enabled middle and high school instructors to explain to elementary school teachers what skills their students would need as they progressed. Frequently teachers at different grade levels were able to help one another understand various concepts. Nor was this assistance limited to the middle and high school teachers helping the elementary school teachers. On the contrary, we found that the middle and high school teachers need the elementary school teachers to explain to them how to solve problems intuitively, i.e., through the use of Cuisenaire Rods and blocks.

Summary

The Mat-Su Math Co-op project was designed to train teachers from the district in improved mathematics techniques, upgrade their math skills, and also to help them incorporate their teaching activities with computer programs and calculators.

The ultimate outcome of this program was to help participants improve their student’s mathematical knowledge and comprehension through reduced anxiety toward mathematics and an increase of students prepared for high school and college mathematics.

Recommendations and Conclusion

The world-wide information superhighway has dramatically increased in its accessibility to the general population. As teachers of mathematics, how do we utilize this vast information bulletin board to help us to teach, to assist us in benefiting our students? The computer, of course, can quickly provide answers, therefore drastically reducing the amount of time spent problem-solving, providing more time for teachers to introduce other new areas to our students.
How can mathematics teachers use the computer to assist in teaching mathematics? This is still a big issue facing us today.

In conclusion, as we analyze and critique our existing teacher training program, we feel the focus of enhancing teacher mathematical skills should encompass the following:

More in-depth training in mathematics

Algebra: Refreshing teachers’ memories in algebra involving applications such as word problems, proportional and distance problems, and introductions to real-life applications of algebra, factoring, and function ideas.

Geometry: Further review in geometry and measurement, with an emphasis on the verification of the identities and proofs of various theorems and conjectures. The teacher should be able to find a counter-example to verify the falsity of the theorem.

Discrete mathematics and applications: Teachers should be encouraged to learn more logic and mathematical evidence as verification, rather than sources for right answers. Teacher training programs should avoid mechanistic and quick-answer techniques so as to teach more mathematical reasoning, procedures for getting an answer, and recursive thinking, which leads to recurrence relations that the given problem can be solved by various techniques, and to verification of solutions by mathematical induction (NCTM, 1991). Combinatorics, the mathematics of counting and arranging objects, should be re-emphasized to teachers, since combinatorial skills are needed in almost every discipline where mathematics is applied, from economics to biology, to computer science to chemistry, to business and management (Epp, 1995).

Teachers also need to be familiar with number theory, such as the word "algorithm." This word was largely unknown three decades ago. Now it is one of the first words encountered in the study of computer science. Actually, the algorithm is a step-by-step method for computers to solve problems. Designing an algorithm requires an understanding of mathematics and the problem to be solved, therefore, the teachers at least have to know how to determine whether or not an algorithm is correct (Epp, 1995).

Teachers also need a basic understanding of probability and statistics, including collecting data, analyzing data, interpreting results, and predicting future trends.

Important goals of discrete mathematics are to develop students’ ability to think abstractly, to learn valid terms of argument, to understand how to communicate effectively with others, and to use both direct and indirect arguments to derive and result from those already known to be true.

These skills should, of course, be re-taught through in-service training programs; however, teachers should also seek mathematics courses at their local college or university, attend math refresher courses for teachers, and utilize the expertise of colleagues in enhancing their skills. Also, the practice of pairing teacher for peer-coaching provides new and inexperienced teachers with the opportunity to learn from a more experienced teacher. This allows inexperienced teachers to learn methods and skills in a non-threatening, non-evaluating manner (Koehler, & Prior, 1993).

Readjust instructional methods
To be an effective teacher of the twenty-first century, the teachers should be encouraged to use a variety of instructional techniques and materials to satisfy the new generations.

**Small group cooperative learning:** Teachers should emphasize the team approach to problem solving and how to work together as members of society.

**Problem solving and applications:** Teachers should use various techniques to help students understand problems and formulate procedures to solve problems. Teachers should re-emphasize that finding the correct answer is not the most important aspect of solving a problem. Rather, teachers must emphasize the importance of cooperation and team work.

**Show-and-tell sessions:** Participants in teacher in-service training programs should need to themselves present mathematics concepts to various groups so the other participants will be able to learn and adapt new information.

**Individualized and self-paced instructions versus classroom lectures:** Teachers need to be aware of techniques, drawbacks, and benefits of individualized and self-paced instructions. Classroom lectures are, of course, necessary; however, relying solely on lectures tends to leave individual students behind. Thus, the in-service teacher must be familiar with both lecture and laboratory methods, utilizing laboratory time to work with individuals on a one-on-one basis.

**Computer assisted and Internet-based instructions:** Teachers need to be computer and Internet literate so as to exploit the growing body of electronic learning materials available. Further, the informational systems which house these materials are becoming more and more a fundamental part of the way students are learning to understand the world around them. In-service teachers must be capable of relating to their students through these electronic informational systems. Thus, they must learn how to utilize this technology within the classroom for the benefit of the institution and the students. This will be one of the most important issues in education for the twenty-first century.

**Relieving math anxiety**

A bi-level strategy must be addressed in dealing with math anxiety. Specifically, these are, first, how to deal with math anxiety experienced by teachers, and, second, how to overcome math anxiety expressed by students.

**Overcoming math anxiety experienced by teachers:** Teacher’s anxiety toward mathematics is largely the result of a poor mathematics background and insufficient mathematics training while in college. In order to remedy their fears in mathematics, they need to actively participate in district-wide in-service refresher courses in basic mathematics or enroll in college level mathematics courses to upgrade their math skills. They should also enroll in courses specifically on "How to Overcome Math Anxiety" whenever possible. Such courses should cover the symptoms of math anxiety, math avoidance, "I can't" syndrome, fun and games in mathematics, and basic revisiting of repeatable testing, math therapy and consultation, and math anxiety scales to test the level of anxiety (Tobias, 1978).

**Overcoming math anxiety expressed by students:** In order to reduce and overcome student’s anxiety toward mathematics, it is important that teachers never suggest that mathematics is too difficult for some people, or act as though only certain special individuals have the capacity to be good at, and
enjoy, mathematics. It is extremely important that teachers display their own enjoyment of mathematics at all times, so as to communicate their enthusiasm to their students.

In addition, teachers should treat testing as an indicator of a student’s weaknesses and strengths rather than as a punitive measure (Chang, 1985), allowing students to retake tests until they understand the material. Retesting gives students assurance that can and do understand mathematics without the fear of a failing grade. When students do make mistakes, it is important that teachers draw the student’s attention to it in a positive manner. For example, the teacher might say, "Johnny, you made a small mistake here. When you try again, I’m sure you’ll do better," or, "Mary, you made a little error here, but as soon as you take another look at it, I’m sure you’ll see how to correct it."

Teachers need to recognize individual student’s optimal learning styles and adapt their teaching method to fit the student’s need while engaging in one-on-one instruction with the student.

Friendly learning climate

The past perception of the teacher as an authoritarian figure is no longer viable. The teacher must now act as a friend to the student, consultant, and facilitator in the learning process. It is important that the teacher always be positive, constantly encouraging students toward a greater understanding and enjoyment of mathematics, providing a helping hand whenever necessary. Given the rapid development of technology, it is important that students experience the human touch of a caring mathematics teacher. This means, of course, no more yelling at students or discouraging them by suggesting that they are incapable of learning mathematics. An evaluative approach to testing and retesting as mentioned above is an important aspect of a friendly learning environment.

A visual aid to understanding the ideal teacher of the twenty-first century and the in-service training programs that they will undergo may be seen in figure three.
* Minimum math literacy skills consist of Algebra, Geometry, and a few topics in Discrete Mathematics such as Probability and Statistics, Set Theory, and Graph Theory

** Bi-monthly all-day retreats should be held at a secluded, private resort, during a weekday other than an in-service day. The school should provide substitute teachers.

*** The school should provide funds to support teachers to attend the workshops and conferences at least on an annual basis.


Conclusion

How can mathematics instructors use the new found scientific learning aids to help them to teach mathematics? This is still a big issue facing us today. How do we teach our new generations of computer-literate students to face more advanced challenges? How often or quickly do we have to re-educate ourselves in order to keep up with changes in mathematics education? How do we move to
distance education without sacrificing traditional educational values? We have to agree, no matter how advanced and complicated technology grows, the need for teachers is still invaluable to our education enterprise in the twenty-first century.

References


ICIUT. Proceedings of the Ninth International Conference for Improving University Teaching, the National Institute for Higher Education in Dublin, Ireland, 1983, pp. 992-1001.


Contact information:

Dr. Ping-Tung Chang                                            Dr. John P. Downes
Professor of Mathematics                                       Professor of Math/Math Education
Matanuska-Susitna College                                      Department of MSIT
University of Alaska Anchorage                                Georgia State University
P.O. Box 2889                                                   Atlanta, Georgia 30303
Palmer, Alaska 99645                                           U.S.A.